1 Conclusion

The dynamics of Gaussian quantum steering and its asymmetry in a two mechanical mode state are examined in an optomechanical setup. This system offers the realistic conditions, which can be implemented experimentally, to illustrate the theoretical formalism of the concept of Gaussian steering [?]. For this end, we have considered a Fabry-Perot double-cavity optomechanical system fed by broadband two-mode squeezed light. We have essentially focused on the quantum steering of the mechanical modes by neglecting the optical cavities modes in the adiabatic regime. In fact the possibility to prepare entangled modes is not only important from conceptual point as well as from applicative perspective. Indeed, the entangled mechanical modes show more robustness against thermal noise which could be of interest for various quantum tasks. To investigate the dynamics of the mechanical modes, we have derived the explicit time-dependent expression of the covariance matrix (Eq. (**??**)) which characterizes the mechanical fluctuations. In analyzing the dynamics of Gaussian steering between the mechanical modes, we have paid a special attention to the situations where the steerability is uni-directional. In this respect, we have shown that one-way steering can be detected in various scenarios from $A \to B$ (see the panel (b) of Fig. **??**) as well as from $B \to A$ (see Fig. **??** and the panels (c), (d) of Fig. **??**. Our analysis demonstrates also revival features of the one-way steering dynamics. (see the panels (c) and (d) of Fig. **??**). For several configurations we have numerically investigated, the one-way steering is in general followed during the evolution of the system by the generation of steerability in both directions. However, it must be noticed that in some exceptional sitautions one can observe the steerability in one direction only (see the panel (d) of Fig. **??**). Therefore, a Fabry-Perot double-cavity optomechanical system offers the adequate experimental scheme for the experimental implementation of Gaussian one-way steering. On the hand, we have shown in the system under consideration that the Gaussian steering is always upper bounded by entanglement and the steering asymmetry $\mathcal{G}^{\Delta}_{AB}$ cannot exceed the value $\ln 2$. This agrees with the results obtained in [?].

Finally, we believe that the Fabry-Perot double-cavity optomechanical system can be of immediate practical interest in detecting the quantum steering and its asymmetry between the entangled mechanical modes. In addition, the optomechanical coupling can be exploited to transfer quantum correlations from optical modes to mechanical modes to gain quantum advantages in implementing long distance quantum protocols. We note also that an equivalent scheme can be considered to investigate gaussian steering between optical modes which may open a new perspectives in the context of quantum key distribution and in quantum information science in general [?, ?, ?].

Einstein-Podolski-Rosen steering is a form of quantum correlations exhibiting an intrinsic asymmetry between two entangled systems. In this paper, we examine the Gaussian steering in a two cavities optomechanical system fed by squeezed light. We work in the resolved sideband regime and we adopt the adiabatic approximation to freeze the optical mode. We show that the Gaussian steering

of the mechanical modes is strongly sensitive to the thermal effect in comparison with entanglement. We verify that Gaussian steering is always upper bounded by entanglement measured by Gaussian Renyi-2 entropy. A specific attention is devoted to the dynamics of unidirectional Gaussian steering under the thermal effect and the degree of squeezing for realistic situations which can be realized experimentally. Other Gaussian steering behaviors are also discussed.