Sudden transition between classical and quantum decoherence

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We study the dynamics of quantum and classical correlations in the presence of nondissipative decoherence. We discover a class of initial states for which the quantum correlations, quantified by the quantum discord, are not destroyed by decoherence for times $t < \bar{t}$. In this initial time interval classical correlations decay. For $t > \bar{t}$, on the other hand, classical correlations do not change in time and only quantum correlations are lost due to the interaction with the environment. Therefore, at the transition time $\bar{t}$ the open system dynamics exhibits a sudden transition from classical to quantum decoherence regime.

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The interaction of a quantum system with its environment causes the rapid destruction of crucial quantum properties, such as the existence of quantum superpositions and of quantum correlations in composite systems [1, 2]. Contrarily to the exponential decay characterizing the transition from a quantum superposition to the corresponding statistical mixture, entanglement may disappear completely after a finite time, an effect known as entanglement sudden death [3]. There exists, however, quantum correlations more general and more fundamental than entanglement. Several measures of these quantum correlations have been investigated in the literature [4–9], and among them the quantum discord [4, 5], has recently received a great deal of attention [10–23].

The total correlations (quantum and classical) in a bipartite quantum system are measured by the quantum mutual information $\mathcal{I}(\rho_{AB})$ defined as

$$\mathcal{I}(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \qquad (1)$$

where $\rho_{A(B)}$ and ρ_{AB} are the reduced density matrix of subsystem $A(B)$ and the density matrix of the total system, respectively, and $S(\rho) = -Tr\{\rho \log_2 \rho\}$ is the von Neumann entropy. The quantum discord is then defined as

$$\mathcal{D}(\rho_{AB}) \equiv \mathcal{I}(\rho_{AB}) - \mathcal{C}(\rho_{AB}) \qquad (2)$$

where $\mathcal{C}(\rho_{AB})$ [see Eq. (3)] are the classical correlations of the state [4, 6, 7]. The quantum discord measures quantum correlations of a more general type than entanglement, there exists indeed separable mixed states having nonzero discord [13]. Interestingly, it has been proven both theoretically and experimentally that such states provide computational speedup compared to classical states in some quantum computation models [13, 24].

The dynamics of quantum and classical correlations in presence of both Markovian [15, 20] and non-Markovian [21] decoherence has been recently investigated. It is believed that the quantum correlations measured by the quantum discord, in the Markovian case, decay exponentially in time and vanish only asymptotically [19, 22], contrarily to the entanglement dynamics where sudden death may occur.

A remarkable result we demonstrate in this Letter is the existence of a class of initial states for which the quantum discord does not decay for a finite time interval $0 < t < \bar{t}$ despite the presence of a noisy environment. Our result is derived for qubits interacting with nondissipative independent reservoirs. It is not yet known whether such phenomenon can be observed for more general types of environment. However, this is the first evidence of the existence of quantum properties, in this case quantum correlations, that remain intact under the action of an open quantum channel.

The major obstacle to the development of quantum technologies has been, until now, the destruction of all quantum properties caused by the inevitable interaction of quantum systems with their environment. The fact that, under certain conditions, quantum correlations useful for quantum algorithms are completely unaffected by the environment, for long time intervals, may constitute a new breakthrough to quantum technologies such as, e.g., quantum computers.

A crucial aspect of the dynamics is that, while the total quantum correlations measured by the discord remain constant, classical correlations are lost. Interestingly, in this dynamical region, entanglement decays exponentially in time but at the same time quantum-correlations-other-than-entanglement, measured by dissonance [9], increase monotonically until $t = \bar{t}$.

We prove analytically that, for certain initial Bell-diagonal states, when discord starts to decay, i.e., for $t > \bar{t}$, the classical correlations become constant in time. Therefore, there exists an instant of time $\bar{t}$ at which the system stops losing classical correlation and starts losing quantum correlations. The time $\bar{t}$ depends on a single parameter characterizing the initial state. The class of initial states for which the sudden transition from quantum to classical decoherence occurs depends on the type of Markovian noise considered.

Let us begin by specifying the quantity used for measuring the classical correlations and, therefore, to calculate the quantum discord by means of Eq. (2). Such a quantity is in fact a second extension of classical mutual information and it is based on the generalization of the concept of conditional entropy. We know that performing measurements on system B affects our knowledge of system A. How much system A is modified by a measurement of B depends on the type of measurement performed on B. Here the measurement is considered of von Neumann type and it is described by a complete set of orthonormal projectors $\{\Pi_k\}$ on subsystem B corresponding to the outcome k. The classical correlations $\mathcal{C}(\rho_{AB})$ are then defined as [4]

$$\mathcal{C}(\rho_{AB}) = \max_{\{\Pi_k\}}[S(\rho_A) - S(\rho_{AB}|\{\Pi_k\})], \qquad (3)$$

where the maximum is taken over the set of projective measurements $\{\Pi_k\}$ and $S(\rho_{AB}|\{\Pi_k\}) = \sum_k p_k S(\rho_k)$ is the conditional entropy of A, given the knowledge of the state of B, with $\rho_k = Tr_B(\Pi_k \rho_{AB} \Pi_k)/p_k$ and $p_k = Tr_{AB}(\rho_{AB}\Pi_k)$.

We consider the case of two qubits under local nondissipative channels, more specifically we focus on phase flip, bit flip and bit-phase flip channels. For each qubit, the Markovian dissipator is given by $\mathcal{L}[\rho_{A(B)}] = \gamma[\sigma_j^{A(B)}\rho_{A(B)}\sigma_j^{A(B)} - \rho_{A(B)}]/2$, with $\sigma_j^{A(B)}$ the Pauli operator in direction j acting on $A(B)$, and $j = 1, 2, 3$ for the bit, bit-phase, and phase flip cases, respectively. For simplicity, we take as initial states of the composite system a class of states with maximally mixed marginals

$$\rho_{AB} = \frac{1}{4}\left(\mathbf{1}_{AB} + \sum_{i=1}^{3} c_i \sigma_i^A \sigma_i^B\right), \qquad (4)$$

where c_i is a real number such that $0 \le |c_i| \le 1$ for every i and $\mathbf{1}_{AB}$ the identity operator of the total system. This class of states includes the Werner states ($|c_1| = |c_2| = |c_3| = c$) and the Bell states $|c_1| = |c_2| = |c_3| = 1$.

We firstly focus on the phase damping (or phase flip) channel. For the initial state of Eq. (4), the time evolution of the total system is given by [15]

$$\begin{aligned}\rho_{AB}(t) = {} & \lambda_\Psi^+(t)|\Psi^+\rangle\langle\Psi^+| + \lambda_\Phi^+(t)|\Phi^+\rangle\langle\Phi^+| \\ & + \lambda_\Phi^-(t)|\Phi^-\rangle\langle\Phi^-| + \lambda_\Psi^-(t)|\Psi^-\rangle\langle\Psi^-|, \qquad (5)\end{aligned}$$

where

$$\lambda_\Psi^\pm(t) = [1 \pm c_1(t) \mp c_2(t) + c_3(t)]/4, \qquad (6)$$
$$\lambda_\Phi^\pm(t) = [1 \pm c_1(t) \pm c_2(t) - c_3(t)]/4, \qquad (7)$$

and $|\Psi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$, $|\Phi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ are the four Bell states. The time dependent coefficients in Eqs. (6)-(7) are $c_1(t) = c_1(0)\exp(-2\gamma t)$, $c_2(t) = c_2(0)\exp(-2\gamma t)$, and $c_3(t) = c_3(0) \equiv c_3$, with γ the phase damping rate.

The mutual information $\mathcal{I}[\rho_{AB}(t)]$ and the classical correlation $\mathcal{C}[\rho_{AB}(t)]$ in this case are given by [8]

$$\mathcal{I}[\rho_{AB}(t)] = 2 + \sum_{k,l} \lambda_k^l(t) \log_2 \lambda_k^l(t), \qquad (8)$$

$$\mathcal{C}[\rho_{AB}(t)] = \sum_{j=1}^{2} \frac{1 + (-1)^j \chi(t)}{2} \log_2[1 + (-1)^j \chi(t)], \qquad (9)$$

where $\chi(t) = \max\{|c_1(t)|, |c_2(t)|, |c_3(t)|\}$, $k = \Psi, \Phi$, and $l = \pm$. We note that the maximization procedure with respect to the projective measurements, present in the definition of the classical correlations of Eq. (3), can be performed explicitly for the system here considered noticing that (i) the complete set of orthogonal projectors is given by $\Pi_j = |\theta_j\rangle\langle\theta_j|$, with $j = 1, 2$, $|\theta_1\rangle = \cos\theta|0\rangle + e^{i\phi}\sin\theta|1\rangle$, $|\theta_2\rangle = e^{-i\phi}\sin\theta|0\rangle - \cos\theta|1\rangle$; and (ii) the state of the system remain always of the form given by Eq. (4) during the time evolution.

We now focus on the class of initial states for which $c_1(0) = \pm 1$ and $c_2(0) = \mp c_3(0)$, with $|c_3| < 1$. These states are mixtures of Bell states of the form

$$\rho_{AB} = \frac{(1 + c_3)}{2}|\Psi^\pm\rangle\langle\Psi^\pm| + \frac{(1 - c_3)}{2}|\Phi^\pm\rangle\langle\Phi^\pm|. \qquad (10)$$

Inserting Eqs. (6)-(7) into Eq. (8) it is straightforward to prove that, for this initial condition, the mutual information takes the form

$$\begin{aligned}\mathcal{I}[\rho_{AB}(t)] = {} & \sum_{j=1}^{2} \frac{1 + (-1)^j c_3}{2} \log_2[1 + (-1)^j c_3] \qquad (11) \\ & + \sum_{j=1}^{2} \frac{1 + (-1)^j c_1(t)}{2} \log_2[1 + (-1)^j c_1(t)].\end{aligned}$$

Having in mind Eq. (9) and remembering that $c_1(t) = \exp(-2\gamma t)$, one sees immediately that, for $t < \bar{t} = -\ln(|c_3|)/(2\gamma)$, the second term in Eq. (11) coincides with the classical correlation $\mathcal{C}[\rho_{AB}(t)]$, since $|c_1(t)| > |c_2(t)|, |c_3(t)| = |c_3|$. The quantum discord is then given by the first term of Eq. (11). Hence, for $t < \bar{t}$, the quantum discord is constant in time. We note that, by changing the initial condition, and in particular $|c_3|$, we can increase the time interval $t < \bar{t}$ over which the discord is constant. For increasing values of $\bar{t}$, however, the quantum discord decreases towards its zero value obtained for $|c_3| = 0$.

In Fig. 1 we plot the time evolution of the quantum discord, the classical correlations and the mutual information for $c_1(0) = 1$, $c_2(0) = -c_3$ and $c_3 = 0.6$. The plot clearly shows the sharp transition from the classical to the quantum decoherence regime occurring at $t = \bar{t}$.

In order to understand the physical origin of the sudden transition from classical to quantum decoherence, we consider the distances between our state and (i) its closest classical state and (ii) its closest separable state. We adopt the definitions proposed in Ref. [9], and we measure all distances by means of the relative entropy. In

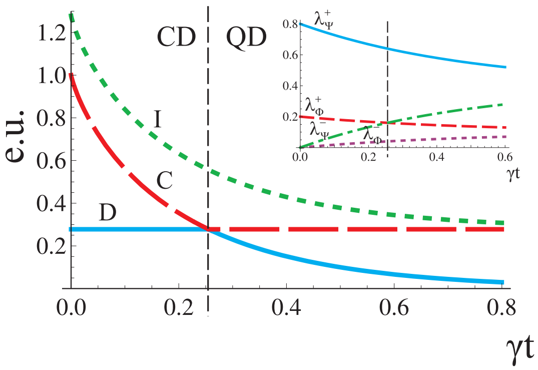

FIG. 1. (Colors online) Dynamics of mutual information (green dotted line), classical correlations (red dashed line) and quantum discord (blue solid line) as a function of γt for $c_1(0) = 1$, $c_2(0) = -c_3$ and $c_3 = 0.6$. In the inset we plot the eigenvalues λ_Ψ^+ (blue solid line), λ_Ψ^- (green dash-dotted line), λ_Φ^+ (red dashed line) and λ_Φ^- (violet dotted line) as a function of γt for the same parameters.

this way, the former distance coincide with a second definition of discord, while the latter distance is the relative entropy of entanglement. We begin by demonstrating that, for the system here considered, the discord defined by Eq. (2) coincides with the one introduced in Ref. [9]. To this aim we notice that, the classical state closest to the state of our system at time t, given by Eq. (5), is [9]

$$\rho_{\rm cl}(t) = \frac{q(t)}{2}\sum_{i=1,2}|\Psi_i\rangle\langle\Psi_i| + \frac{1-q(t)}{2}\sum_{i=3,4}|\Psi_i\rangle\langle\Psi_i|, \quad (12)$$

with $q(t) = \lambda_1(t) + \lambda_2(t)$, where $\lambda_1(t)$ and $\lambda_2(t)$ are the two highest eigenvalues given by Eqs. (6)-(7), and $|\Psi_i\rangle$ the corresponding Bell states. In the inset of Fig. 1 we plot the eigenvalues $\lambda_\Psi^\pm$ and $\lambda_\Phi^\pm$, giving the weights or populations of the four Bell states components. The inset shows that at $t = \bar{t}$ the population of $|\Phi_+\rangle$ becomes equal to the population of $|\Psi_-\rangle$ and, subsequently, it continues to decrease while the other one grows. As a consequence of this switch in the second highest population component, for $t < \bar{t}$, the closest classical state is

$$\begin{aligned}\rho_{\rm cl}(t<\bar{t}) = &\frac{1+e^{-2\gamma t}}{4}\left(|\Psi^+\rangle\langle\Psi^+| + |\Phi^+\rangle\langle\Phi^+|\right)\\ &+\frac{1-e^{-2\gamma t}}{4}\left(|\Phi^-\rangle\langle\Phi^-| + |\Psi^-\rangle\langle\Psi^-|\right), \quad (13)\end{aligned}$$

while, for $t > \bar{t}$,

$$\begin{aligned}\rho_{\rm cl}(t>\bar{t}) = &\frac{1+c_3}{4}\left(|\Psi^+\rangle\langle\Psi^+| + |\Psi^-\rangle\langle\Psi^-|\right)\\ &+\frac{1-c_3}{4}\left(|\Phi^-\rangle\langle\Phi^-| + |\Phi^+\rangle\langle\Phi^+|\right). \quad (14)\end{aligned}$$

Let us now look at the dynamics of the relative entropy $D(\rho_{AB}\|\rho_{\rm cl}) = -Tr\{\rho_{AB}\log_2\rho_{\rm cl}\} + Tr\{\rho_{AB}\log\rho_{AB}\}$ [9]. Inserting Eq. (5) and Eqs. (13)-(14) into the expression for $D(\rho_{AB}\|\rho_{\rm cl})$, it is straightforward to prove that $D(\rho_{AB}\|\rho_{\rm cl}) = \mathcal{D}(\rho_{AB})$. This result holds for all the states of the form of Eq. (4). Hence, in the first dynamical regime, when the discord is constant and only classical correlations are lost, the distance to the closest classical state remains constant. At $t = \bar{t}$, the closest classical state changes suddenly from the one given by Eq. (13) to the one given by Eq. (14). Subsequently, for $t > \bar{t}$, $\rho_{\rm cl}$ remains constant in time and the state of the system approaches asymptotically such state, as indicated by the monotonic decay of the quantum discord. This behavior suggests a sufficient condition for the occurrence of the sudden transition between classical and quantum decoherence. This transition is present in the dynamics for those classes of initial states and dynamical maps for which (i) the state is at all times of the form of Eq. (4) and (ii) its distance to the closest classical state is constant.

To further understand the dynamics of the total quantum correlations, we study the relative entropy of entanglement E and the dissonance Q defined as the distance to the closest separable state ρ_S and the distance between ρ_S and its closest classical state ρ_{SC}, respectively [9]. Both quantities can be calculated exactly in our model. Entanglement takes the simple form $E = 1 + \lambda_1\log_2\lambda_1 + (1-\lambda_1)\log_2(1-\lambda_1)$, with λ_1 the highest of the eigenvalues given by Eqs. (6)-(7). This equation shows that entanglement always decays monotonically and it vanishes completely for $t \geq t_S = -\ln[(1-|c_3|)/(1+|c_3|)]/(2\gamma)$. This result is independent of the entanglement measure since all entanglement measures coincide and are equal to zero for separable states. If $t_S < \bar{t}$, then entanglement disappears completely when the quantum discord has not yet started to decay so the state of the total system is a separable state with nonzero discord. These are the states exploited in the one-qubit model of quantum computation of Ref. [13]. One can easily check that $t_S < \bar{t}$ whenever $0 < |c_3| < \sqrt{2}-1$. Figure 2 shows one of such examples. Moreover, there exist classes of initial separable states for which discord remains constant for $t < \bar{t}$ while entanglement is always zero. It is simple to see by direct substitution that, e.g., the state of the form of Eq. (4) with $c_1 = \pm(1-|c_3|)/(1+|c_3|)$, $c_2 = -c_3(1-|c_3|)/(1+|c_3|)$, and $0 < |c_3| < \sqrt{2}-1$ displays this behavior.

Let us, finally, look at the dissonance. We obtain $Q = 1 + \sum_{i=1}^4 p_i\log_2 p_i - (p_1+p_2)\log_2(p_1+p_2) + (1-p_1-p_2)\log_2(1-p_i-p_2)$, with $p_1 = 1/2$, $p_i = \lambda_i/2(1-\lambda_1)$, and $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ the eigenvalues of Eqs. (6)-(7) in non increasing order. Figure 3 shows the time evolution of discord, entanglement and dissonance, all measured in entropic units. Remarkably, while entanglement decays, dissonance increases monotonically in time until $t = \bar{t}$. This means that while the state of the system approaches its closest separable state, this state in turns goes farther and farther from its closest classical state. The increase in the dissonance indicates an in-

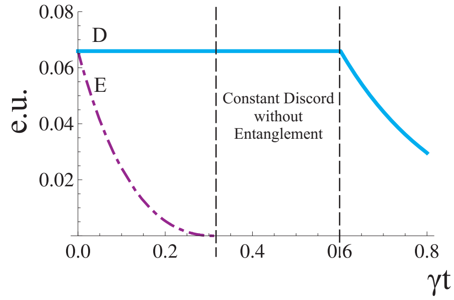

FIG. 2. (Colors online) Dynamics of entanglement (violet dashed-dotted line) and quantum discord (blue solid line) as a function of γt for $c_1(0) = 1$, $c_2(0) = -c_3$ and $c_3 = 0.3$.

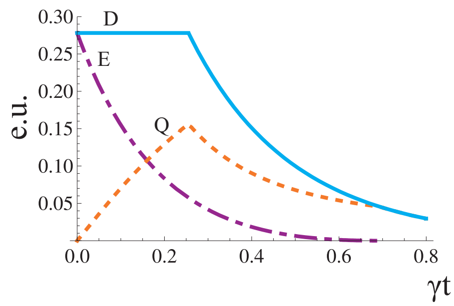

FIG. 3. Dynamics of entanglement (violet dashed-dotted line), quantum discord (blue solid line) and dissonance (orange dashed line) as a function of γt for $c_1(0) = 1$, $c_2(0) = -c_3$ and $c_3 = 0.6$.

crease in other-than-entanglement quantum correlations which contribute to mantain the total quantum correlations (discord) constant. It is worth noticing, however, that, as noted in Ref. [9], dissonance and entanglement do not add to give the discord because of the subadditivity of correlations. It is simple to see that, for the bit flip and phase-bit flip channels, the class of states for which the sudden transition from classical to quantum decoherence occurs, have the same form of Eq. (10), with c_1 and c_2 replacing c_3, respectively.

The existence of a sharp transition between classical and quantum loss of correlations in a composite system is a remarkable feature of the dynamics of composite open quantum system that was up to now unknown. The existence of a finite time interval during which quantum correlations initially present in the state do not decay in presence of decoherence opens a series of interesting questions. Is it possible to exploit the class of initial states displaying such a property to perform quantum computation or communication tasks without any disturbance from the noisy environment for long enough intervals of time? Which is the most general class of states and of open quantum systems exhibiting a sudden transition from classical to quantum decoherence? Finally, and perhaps most importantly, which are the physical mechanisms that forbid the loss of quantum correlations at the initial times and that allow only quantum correlations to be lost after the transition time $\bar{t}$? We believe that the transition from classical to quantum decoherence presented in this Letter and very recently confirmed experimentally [25] will shed new light on one of the most fundamental and fascinating aspects of quantum theory.

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