Quantum uncertainty and quantum phase transition in spin chains with Dzyaloshinski-Moriya interacion

N. Habiballah [a1] and **M. Daoud**[b,c 2]

[a]*Department of Physics, Campus Universitaire Ait Melloul, University Ibnou Zohr, Agadir , Morocco*

[b]*Department of Physics , Faculty of Sciences Ain Chock, University Hassan II, Casablanca, Morocco*

[c]*Abdus Salam International Centre for Theoretical Physics,Strada Costiera, 11 I - 34151, Trieste, Italy*

Abstract

We employ the local quantum uncertainty (LQU) as reliable quantifier of discord-like correlations of a two-qubit one dimensional XYZ Heisenberg chain with Dzyaloshinski-Moriya interaction, in thermal equilibrium. We discuss the behavior of thermal quantum discord in terms of the temperature of the bath, the strength of the external magnetic fields. A special emphasis is devoted to the effects induced by the Dzyaloshinski-Moriya interaction.

[1]email: n.habiballah@uiz.ac.ma
[2]email: m_daoud@hotmail.com

1 Introduction

Quantum correlations in finite dimensional quantum systems are widely recognized as crucial resources to enhance the performance of quantum protocols in comparison with their classical analogue [1]. However, from applicative point of view, the exploitation of quantum correlations is limited by the decoherence effects resulting from the system-environment interaction [2]. Various strategies to annihilate or at least to reduce the effects of environmental couplings in open-system evolutions were proposed in the literature. On the other hand, from a fundamental point of view, considerable effort to distinguish the main features differentiating between classical and non classical correlations. This explains the extensive investigations devoted to the quantification of quantum correlations in quantum systems comprising two or more qubits. While pure states can be separable or entangled, the mixed states exhibit more subtle features of non-classical correlations. In this sense, various quantum correlations indicators were investigated during the last two-decades. Each quantifier presents advantages and disadvantages. For instance, the quantum discord introduced in [3, 4] to characterize quantum correlations beyond the entanglement is not easily computable for a generic two qubit state. The geometric quantum discord based on Hilbert-Schmidt [5] is appropriate for calculations but grow under local operations on unmeasured qubit and therefore can not be considered as a faithful indicator of quantmness [6, 7]. This leads to a reformulation of geometric quantum discord by employing the trace distance (Schatten 1-norm) [8, 9, 10, 11]. These various measures have provided the tools to gain deeper understanding of the role of quantum correlations in many-body systems and especially in quantum phase transitions [12, 13, 14]. The changes in quantum correlations affect strongly the properties of a many-body system. Interesting results were obtained in analyzing the connection between the pairwise entanglement and quantum phase transitions in quantum spin chains [15, 16]. Also, the pairwise quantum discord has been shown more adequate to reveal quantum phase transitions in such systems even in the absence of entanglement [17, 18]. This is essentially due to the robustness character of this quantum quantifier introduced initially to go beyond entanglement measure. The interplay between quantum phase transitions and quantum correlations at finite temperatures was also considered in some recent works (see for instance [19, 20]). Quantum phase transitions occur at absolute zero temperature which cannot be reached experimentally. Therefore the detection of quantum phase transitions require to work at very low temperatures where the quantum fluctuations are dominant.

Besides the aforementioned measures, the local quantum uncertainty [21] was recently adopted to detect the occurrence of quantum criticality in multipartite spin systems [22, 23]. The local quantum uncertainty is a quantum discord-like defined as the coherence based measure of quantum correlations induced by the Wigner-Yanase skew information [24]. The local quantum uncertainty all the properties that a bona fide indicator is required to exhibit [21]. In addition, this reliable quantifier is effortlessly computable contrarily to some other measures for which closed analytical expressions are not always easy obtainable. We note also that the local quantum uncertainty is related to the Fisher information and therefore constitutes a key tool in quantum metrology protocols [25]. In this paper we shall employ the local quantum uncertainty as a quantifier of discord-like correlations in a two qubit one dimensional XYZ Heisenberg spin chain in nonuniform external field with

Dzyaloshinski-Moriya interaction [26, 27] which arises from the spin-orbit coupling. We notice that in the last decade, there has been an ongoing effort to quantify the entanglement and quantum discord in such system to understand the effects of spin-spin interaction and spin-orbit coupling on the entanglement properties. Several scenarios were investigated to point out the role of Dzyaloshinski-Moriya interaction in protecting or suppressing the quantum correlations in in two-qubit Heisenberg systems [28, 29, 30, 31, 32, 33]. In this sense, the main goal of this work is the quantification of quantum correlations in XYZ Heisenberg model via the notion of local quantum uncertainty and to characterize the corresponding behaviors with respect to the effects induced by the Dzyaloshinski-Moriya coupling in different magnetic field regimes.

The plan of the paper is as follows. In section 2, we first introduce the thermal density matrix describing a two qubit system with Dzyaloshinski-Moriya interaction in equilibrium with a thermal bath. We also give the analytical expression of local quantum uncertainty measuring the amount of quantum correlations contained in the system. To analyze the effects of the Dzyaloshinski-Moriya interaction on the quantum discord we shall consider in section 3 some special cases of the XYZ Heisenberg model, namely Ising Model, isotropic Heisenberg model and some variants of XYZ spin models which are tractable numerically. A particular attention is also devoted to the comparison of local quantum uncertainty and entanglement of formation to show that this new kind of quantumness indicator goes indeed beyond entanglement. Concluding remarks close this paper.

2 Thermal quantum discord via local quantum uncertainty

It is well known that the XYZ Heisenberg models describe appropriately magnetic properties in solids. It is also adapted to the investigation of the connection between quantum correlations and quantum phase transitions. Hence, we shall study the amount of local quantum uncertainty in a pair of qubits (spin-1/2) at finite temperature in presence of an external magnetic fields acting on both qubits. We also consider the situation where the two qubits are coupled via the Dzyaloshinski-Moriya interaction which arises from the spin-orbit coupling. It can be described by

$$H_{\rm DM} \sim \overrightarrow{D}.(\overrightarrow{\sigma}_1 \wedge \overrightarrow{\sigma}_2)$$

where $\overrightarrow{D} = (D_x, D_y, D_z)$ denotes the vector coupling and σ_i^x, σ_i^y and σ_i^z are the usual Pauli matrices associated with the two qubits ($i = 1, 2$). In what follows we shall restrict our study to the situation where the Dzyaloshinski-Moriya interaction is along the z direction (i.e, $D_x = D_y = 0$).

2.1 The Heisenberg model with Dzyaloshinski-Moriya interaction

In the framework of XYZ Heisenberg model, the Hamiltonian describing two spin-1/2 with Dzyaloshinski-Moriya interaction along the z-direction, is given by

J'ai pose $B_z = b_1 + b_2$ et $b_z = b_1 - b_2$

$$H = -J_x\sigma_1^x\sigma_2^x - J_y\sigma_1^y\sigma_2^y - J_z\sigma_1^z\sigma_2^z - \frac{1}{2}D_z(\sigma_1^x\sigma_2^y - \sigma_1^y\sigma_2^x) - b_1\sigma_1^z - b_2\sigma_2^z \tag{1}$$

where J_i $(i = x, y, z)$ denote the coupling constants, b_1 (resp. b_2) is the z-component of the external magnetic field acting on the qubit 1 (resp. qubit 2). For $J_i > 0$ (resp. $J_i < 0$) corresponds to the ferromagnetic (resp.

anti-ferromagnetic) phase. In the two-qubit computational basis $\mathcal{B}$={$|0,0\rangle$, $|0,1\rangle$, $|1,0\rangle$, $|1,1\rangle$}, the Hamiltonian (1) can be expressed as following form:

$$H = \begin{pmatrix} -b_1 - b_2 - J_z & 0 & 0 & -J_x + J_y \\ 0 & b_1 - b_2 + J_z & -J_x - J_y - iD_z & 0 \\ 0 & -J_x - J_y + iD_z & -b_1 + b_2 + J_z & 0 \\ -J_x + J_y & 0 & 0 & b_1 + b_2 - J_z \end{pmatrix} \tag{2}$$

The corresponding eigenvalues and eigenstates are given by

$$E_1 = -J_z + \sqrt{(b_1+b_2)^2 + (J_-)^2}, \qquad |\varphi_1\rangle = \cos\left(\frac{\theta_1}{2}\right)|00\rangle + \sin\left(\frac{\theta_1}{2}\right)|11\rangle, \tag{3}$$

$$E_2 = -J_z - \sqrt{(b_1+b_2)^2 + (J_-)^2}, \qquad |\varphi_2\rangle = \sin\left(\frac{\theta_1}{2}\right)|00\rangle - \cos\left(\frac{\theta_1}{2}\right)|11\rangle, \tag{4}$$

$$E_3 = J_z + \sqrt{(b_1-b_2)^2 + D_z^2 + (J_+)^2}, \qquad |\varphi_3\rangle = \cos\left(\frac{\theta_2}{2}\right)|01\rangle + e^{-i\varphi}\sin\left(\frac{\theta_2}{2}\right)|10\rangle, \tag{5}$$

$$E_4 = J_z - \sqrt{(b_1-b_2)^2 + D_z^2 + (J_+)^2}, \qquad |\varphi_4\rangle = \sin\left(\frac{\theta_2}{2}\right)|01\rangle - e^{-i\varphi}\cos\left(\frac{\theta_2}{2}\right)|10\rangle, \tag{6}$$

where $J_\pm = J_x \pm J_y$, $\tan(\theta_1) = -\frac{J_-}{b_1+b_2}$, $\tan(\theta_2) = \frac{\sqrt{D_z^2+(J_+)^2}}{b_1+b_2}$ and $\tan(\varphi) = \frac{D_z}{J_+}$.

The system is assumed to be in equilibrium with a thermal reservoir at temperature T (canonical ensemble). It is then described by the density operator

$$\rho(T) = \frac{e^{-\beta H}}{Z} = \frac{1}{Z}\sum_{i=1}^{4} e^{-\beta E_i}|\varphi_i\rangle\langle\varphi_i| \tag{7}$$

where $Z = \mathrm{Tr}e^{-\beta H}$ is the partition function of the system and $\beta = 1/k_B T$ (k_B is the Boltzmann constant). Reporting (3), (4), (5) and (6) in the equation (7), the thermal state $\rho(T)$ writes, in the computational basis, as

$$\rho(T) = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix} \tag{8}$$

where the diagonal entries are given by

$$\rho_{11} = \frac{e^{\beta J_z}}{Z}\left[e^{-\beta\Delta}\cos^2\left(\frac{\theta_1}{2}\right) + e^{\beta\Delta}\sin^2\left(\frac{\theta_1}{2}\right)\right], \quad \rho_{22} = \frac{e^{-\beta J_z}}{Z}\left[e^{-\beta\Delta'}\cos^2\left(\frac{\theta_2}{2}\right) + e^{\beta\Delta'}\sin^2\left(\frac{\theta_2}{2}\right)\right], \tag{9}$$

$$\rho_{33} = \frac{e^{-\beta J_z}}{Z}\left[e^{-\beta\Delta'}\sin^2\left(\frac{\theta_2}{2}\right) + e^{\beta\Delta'}\cos^2\left(\frac{\theta_2}{2}\right)\right], \quad \rho_{44} = \frac{e^{\beta J_z}}{Z}\left[e^{-\beta\Delta}\sin^2\left(\frac{\theta_1}{2}\right) + e^{\beta\Delta}\cos^2\left(\frac{\theta_1}{2}\right)\right], \tag{10}$$

and the off-diagonal matrix elements are

$$\rho_{14} = \rho_{41} = -\frac{e^{\beta J_z}}{Z}\sin(\theta_1)\sinh(\beta\Delta), \qquad \rho_{23} = \bar{\rho}_{32} = -\, e^{i\varphi}\frac{e^{-\beta J_z}}{Z}\sin(\theta_2)\sinh(\beta\Delta'), \tag{11}$$

in terms of the quantities Δ and Δ' defined by $\Delta = \sqrt{(b_1+b_2)^2 + J_-^2}$, and $\Delta' = \sqrt{(b_1-b_2)^2 + D_z^2 + J_+^2}$. The partition function Z is given by the following expression

$$Z = 2\, e^{\beta J_z}\cosh(\beta\Delta) + 2\, e^{-\beta J_z}\cosh(\beta\,\Delta')$$

2.2 Local quantum uncertainty

Due to its easiness of computability, the local quantum uncertainty is now considered as a promising quantifier of quantum correlation in multipartite systems. For pure bipartite states, it reduces to linear entropy of the reduced densities of the subsystems. Also, it vanishes for classically correlated states. Another interesting property of local quantum uncertainty is its invariance under local unitary operations. This quantumness indicator enjoys all required properties of being a reliable quantifier of quantum correlations [21]. The local quantum uncertainty quantifies the minimal quantum uncertainty in a quantum state due to a measurement of a local observable [21]. For a bipartite quantum state ρ, the local quantum uncertainty is defined as

$$\mathcal{U}(\rho) \equiv \min_{K_1} \mathcal{I}(\rho, K_1 \otimes \mathbb{I}_2), \tag{12}$$

where K_1 is some local observable on the qubit 1, $\mathbb{I}_2$ is the identity operator and

$$\mathcal{I}(\rho, K_1 \otimes \mathbb{I}_2) = -\frac{1}{2}\mathrm{Tr}([\sqrt{\rho}, K_1 \otimes \mathbb{I}_2]^2) \tag{13}$$

is the skew information [24, 34]. The skew information represents the non-commutativity between the state and the observable K_1. The analytical evaluation the local quantum uncertainty requirers a minimization procedure over the set of all observales acting on the part 1. A closed form for qubit-qudit systems was derived in [21]. In particular, for qubits ($\frac{1}{2}$-spin particles), the expression of the local quantum uncertainty is given by [21]

$$\mathcal{U}(\rho) = 1 - \max\{\lambda_1, \lambda_2, \lambda_3\}, \tag{14}$$

where λ_1, λ_2 and λ_3 are the eigenvalues of the 3×3 matrix W whose matrix elements are defined by

$$\omega_{ij} \equiv \mathrm{Tr}\{\sqrt{\rho}(\sigma_i \otimes \mathbb{I}_2)\sqrt{\rho}(\sigma_j \otimes \mathbb{I}_2)\}, \tag{15}$$

with $i, j = 1, 2, 3$.

The thermal state (28) is X-shaped. This resembles to the alphabet X with non-zero entries only along the diagonal and anti-diagonal. The Fano-Bloch decomposition of the state $\rho(T)$ writes as

$$\rho(T) = \frac{1}{4}\sum_{\alpha,\beta} R_{\alpha\beta}\sigma_\alpha \otimes \sigma_\beta$$

where the correlation matrix $R_{\alpha\beta}$ are given by $R_{\alpha\beta} = \mathrm{Tr}(\sqrt{\rho}\ \sigma_\alpha \otimes \sigma_\beta)$. They write

$$R_{03} = 1 - 2\rho_{22} - 2\rho_{44}, \quad R_{30} = 1 - 2\rho_{33} - 2\rho_{44}, \quad R_{11} = 2\ \mathrm{Re}(\rho_{32} + \rho_{41}), \quad R_{22} = 2\ \mathrm{Re}(\rho_{32} - \rho_{41})$$

$$R_{12} = -2i\ \mathrm{Im}(\rho_{41} - \rho_{32}) \quad R_{21} = -2i\ \mathrm{Im}(\rho_{41} + \rho_{32}), \quad R_{00} = \rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1, \quad R_{33} = 1 - 2\rho_{22} - 2\rho_{33}.$$

Calculer les elements $R_{\alpha\beta}$ pour la matrice $\rho(T)$

It is simple to check that the square root of the density matrix $\rho(T)$ is also X-shaped and writes as

$$\sqrt{\rho(T)} = \begin{pmatrix} \tilde{\rho}_{11} & 0 & 0 & \tilde{\rho}_{14} \\ 0 & \tilde{\rho}_{22} & \tilde{\rho}_{23} & 0 \\ 0 & \tilde{\rho}_{32} & \tilde{\rho}_{33} & 0 \\ \tilde{\rho}_{41} & 0 & 0 & \tilde{\rho}_{44} \end{pmatrix} \tag{16}$$

where the diagonal entries are given by

$$\tilde{\rho}_{11} = \sqrt{\frac{e^{\beta J_z}}{2Z}} \left(\frac{1 + e^{-\beta\Delta} + 2\sin^2\left(\frac{\theta_1}{2}\right)\sinh\left(\beta\Delta\right)}{\sqrt{1+\cosh(\beta\Delta)}} \right), \; \tilde{\rho}_{22} = \sqrt{\frac{e^{-\beta J_z}}{2Z}} \left(\frac{1 + e^{-\beta\Delta'} + 2\sin^2\left(\frac{\theta_2}{2}\right)\sinh\left(\beta\Delta'\right)}{\sqrt{1+\cosh(\beta\Delta')}} \right), \quad (17)$$

$$\tilde{\rho}_{33} = \sqrt{\frac{e^{-\beta J_z}}{2Z}} \left(\frac{1 + e^{\beta\Delta'} - 2\sin^2\left(\frac{\theta_2}{2}\right)\sinh\left(\beta\Delta'\right)}{\sqrt{1+\cosh(\beta\Delta')}} \right), \; \tilde{\rho}_{44} = \sqrt{\frac{e^{\beta J_z}}{2Z}} \left(\frac{1 + e^{\beta\Delta} - 2\sin^2\left(\frac{\theta_1}{2}\right)\sinh\left(\beta\Delta\right)}{\sqrt{1+\cosh(\beta\Delta)}} \right), \quad (18)$$

and the off-diagonal matrix elements are

$$\tilde{\rho}_{14} = \tilde{\rho}_{41} = -\sqrt{\frac{e^{\beta J_z}}{2Z}} \left(\frac{\sin\left(\theta_1\right)\sinh\left(\beta\Delta\right)}{\sqrt{1+\cosh(\beta\Delta)}} \right), \qquad \tilde{\rho}_{23} = \overline{\tilde{\rho}_{32}} = -\sqrt{\frac{e^{-\beta J_z}}{2Z}} \left(\frac{e^{i\varphi}\sin\left(\theta_2\right)\sinh\left(\beta\Delta'\right)}{\sqrt{1+\cosh(\beta\Delta')}} \right). \quad (19)$$

In Fano-Bloch representation, the matrix $\sqrt{\rho(T)}$ writes as

$$\sqrt{\rho(T)} = \frac{1}{4}\sum_{\alpha,\beta} \mathcal{R}_{\alpha\beta}\sigma_\alpha \otimes \sigma_\beta$$

with $\mathcal{R}_{\alpha\beta} = \mathrm{Tr}(\sqrt{\rho(T)}\ \sigma_\alpha \otimes \sigma_\beta)$. The non vanishing matrix correlation elements $\mathcal{R}_{\alpha\beta}$ are explicitly given by

$$\mathcal{R}_{00} = \sqrt{t_1 + 2\sqrt{d_1}} + \sqrt{t_2 + 2\sqrt{d_2}} \qquad \mathcal{R}_{03} = \frac{1}{2}\frac{R_{30}+R_{03}}{\sqrt{t_1 + 2\sqrt{d_1}}} - \frac{1}{2}\frac{R_{30}-R_{03}}{\sqrt{t_2 + 2\sqrt{d_2}}}$$

$$\mathcal{R}_{30} = \frac{1}{2}\frac{R_{30}+R_{03}}{\sqrt{t_1 + 2\sqrt{d_1}}} + \frac{1}{2}\frac{R_{30}-R_{03}}{\sqrt{t_2 + 2\sqrt{d_2}}} \qquad \mathcal{R}_{11} = \frac{1}{2}\frac{R_{11}+R_{22}}{\sqrt{t_2 + 2\sqrt{d_2}}} + \frac{1}{2}\frac{R_{11}-R_{22}}{\sqrt{t_1 + 2\sqrt{d_1}}}$$

$$\mathcal{R}_{12} = \frac{1}{2}\frac{R_{12}+R_{21}}{\sqrt{t_1 + 2\sqrt{d_1}}} + \frac{1}{2}\frac{R_{12}-R_{21}}{\sqrt{t_2 + 2\sqrt{d_2}}} \qquad \mathcal{R}_{21} = \frac{1}{2}\frac{R_{12}+R_{21}}{\sqrt{t_1 + 2\sqrt{d_1}}} - \frac{1}{2}\frac{R_{12}-R_{21}}{\sqrt{t_2 + 2\sqrt{d_2}}}$$

$$\mathcal{R}_{22} = \frac{1}{2}\frac{R_{11}+R_{22}}{\sqrt{t_2 + 2\sqrt{d_2}}} - \frac{1}{2}\frac{R_{11}-R_{22}}{\sqrt{t_1 + 2\sqrt{d_1}}} \qquad \mathcal{R}_{33} = \sqrt{t_1 + 2\sqrt{d_1}} - \sqrt{t_2 + 2\sqrt{d_2}}$$

where $t_1 = \rho_{11} + \rho_{44}$, $t_2 = \rho_{22} + \rho_{33}$, $d_1 = \rho_{11}\rho_{44} - \rho_{14}\rho_{41}$ and $d_2 = \rho_{22}\rho_{33} - \rho_{23}\rho_{32}$.

Calculer les elements $\mathcal{R}_{\alpha\beta}$ pour la matrice $\sqrt{\rho(T)}$

At this stage, we have the tools to evaluate the matrix elements defined by

$$\omega_{ij} = \mathrm{Tr}\Big(\sqrt{\rho(T)}\ (\sigma_i \otimes \sigma_0)\sqrt{\rho(T)}\ (\sigma_j \otimes \sigma_0)\Big)$$

where i and j take the values 1, 2, 3. Using the following identities

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij} \quad \mathrm{Tr}(\sigma_i\sigma_j) = 2\delta_{ij} \quad \mathrm{Tr}(\sigma_i\sigma_j\sigma_k\sigma_l) = 2(\delta_{ij}\delta_{kl} - \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}),$$

The elements ω_{ij} write

$$\omega_{ij} = \delta_{ij}\left[\frac{1}{4}\sum_\beta\left(\mathcal{R}_{0\beta}^2 - \sum_k \mathcal{R}_{k\beta}^2\right)\right] + \frac{1}{2}\sum_\beta \mathcal{R}_{i\beta}\mathcal{R}_{j\beta}$$

where $\beta = 0, 1, 2, 3$ and $k = 1, 2, 3$. The diagonal elements are

$$\omega_{ii} = \frac{1}{4}\left[\sum_\beta\left(\mathcal{R}_{0\beta}^2 - \sum_k \mathcal{R}_{k\beta}^2\right)\right] + \frac{1}{2}\sum_\beta \mathcal{R}_{i\beta}^2$$

and the off-diagonal elements are

$$\omega_{ij} = \frac{1}{2}\sum_\beta \mathcal{R}_{i\beta}\mathcal{R}_{j\beta} \quad i \neq j$$

Explicitly, we have

$$\omega_{11} = \frac{1}{4}\Big[(\mathcal{R}_{00}^2 - \mathcal{R}_{33}^2) + (\mathcal{R}_{11}^2 - \mathcal{R}_{22}^2) + (\mathcal{R}_{12}^2 - \mathcal{R}_{21}^2) + (\mathcal{R}_{03}^2 - \mathcal{R}_{30}^2)\Big] \tag{20}$$

$$\omega_{22} = \frac{1}{4}\Big[(\mathcal{R}_{00}^2 - \mathcal{R}_{33}^2) + (\mathcal{R}_{22}^2 - \mathcal{R}_{11}^2) + (\mathcal{R}_{21}^2 - \mathcal{R}_{12}^2) + (\mathcal{R}_{03}^2 - \mathcal{R}_{30}^2)\Big] \tag{21}$$

$$\omega_{33} = \frac{1}{4}\Big[(\mathcal{R}_{00}^2 + \mathcal{R}_{33}^2) - (\mathcal{R}_{11}^2 + \mathcal{R}_{22}^2) - (\mathcal{R}_{12}^2 + \mathcal{R}_{21}^2) + (\mathcal{R}_{03}^2 + \mathcal{R}_{30}^2)\Big] \tag{22}$$

$$\omega_{12} = \omega_{21} = \frac{1}{2}\big(\mathcal{R}_{11}\mathcal{R}_{21} + \mathcal{R}_{12}\mathcal{R}_{22}\big), \qquad \omega_{13} = \omega_{31} = 0, \qquad \omega_{23} = \omega_{32} = 0. \tag{23}$$

Calculer les elements ω_{ij} et diagonaliser la matrice W

The eigenvalues of the matrix W (cf. 15) write

$$\lambda_1 = \frac{1}{2}(\omega_{11} + \omega_{22}) + \sqrt{\omega_{12}^2 + \frac{1}{4}(\omega_{11} - \omega_{22})^2} \tag{24}$$

$$\lambda_2 = \frac{1}{2}(\omega_{11} + \omega_{22}) - \sqrt{\omega_{12}^2 + \frac{1}{4}(\omega_{11} - \omega_{22})^2} \tag{25}$$

$$\lambda_3 = \omega_{33}. \tag{26}$$

It is clear that $\lambda_2 \leq \lambda_2$ and as result the LQU (12) is then given by

$$U(\rho) = 1 - \max\{\lambda_1, \lambda_3\} \tag{27}$$

Verifier que ces resultats conduisent aux valeurs propres que tu as calcule pour la matrice W donnees dans ce qui suit

The explicit form of the eigenvalues of λ_1, λ_2 and λ_3 are given by

$$\begin{aligned}
\lambda_1 &= \frac{2}{Z}\left(\frac{1+\cosh(\beta\Delta') + \cosh(\beta\Delta) + \cosh\beta(\Delta-\Delta') + 2\sinh(\beta\Delta')\sinh(\beta\Delta)\sin^2\left(\frac{\theta_1+\theta_2}{2}\right)}{\sqrt{1+\cosh(\beta\Delta)}\sqrt{1+\cosh(\beta\Delta')}}\right)\\
\lambda_2 &= \frac{2}{Z}\left(\frac{1+\cosh(\beta\Delta') + \cosh(\beta\Delta) + \cosh\beta(\Delta-\Delta') + 2\sinh(\beta\Delta')\sinh(\beta\Delta)\sin^2\left(\frac{\theta_1-\theta_2}{2}\right)}{\sqrt{1+\cosh(\beta\Delta)}\sqrt{1+\cosh(\beta\Delta')}}\right),\\
\lambda_3 &= \frac{e^{\beta J_z}}{Z}\left(\frac{1+2\cosh(\beta\Delta) + \cosh(2\beta\Delta) - 2\sin^2\theta_1\sinh^2\beta\Delta}{1+\cosh(\beta\Delta)}\right)\\
&\qquad + \frac{e^{-\beta J_z}}{Z}\left(\frac{1+2\cosh(\beta\Delta') + \cosh(2\beta\Delta') - 2\sin^2(\theta_2)\sinh^2(\beta\Delta')}{1+\cosh(\beta\Delta')}\right).
\end{aligned} \tag{28}$$

Therefore one should consider the situations where $\lambda_{\max}$ is λ_1 or λ_3. This question is examined numerically in what for various values of the coupling constants J_x, J_y and J_z, in presence or absence of external magnetic fields as well as the effect of DM interaction on the quantum correlations in the state $\rho(T)$.

3 The effect of DM interaction on thermal quantum discord

Using the analytical formula (27) for the local quantum uncertainty, we present a detailed study for the the general XYZ model. Before to to this, we shall consider first consider the Ising model ($J_x = J_y = 0$), the XY model ($J_z = 0$), the XXX model ($J_x = J_y = J_z = J$) and the XXZ ($J_x = J_y = J, J_z \neq J$). For each of the above mentioned models we discuss the quantum discord in presence or absence of magnetic fields. We especially focus on the behavior of quantum correlations versus the temperature and the DM interaction coupling.

3.0.1 Ising Model (J_x=$J_y = 0$ and $J_z \neq 0$)

Premier cas $(i) b_1 = b_2 = 0$

Prendre une valeur positive J_z (0.5) et aussi une valeur negative (-0.5)

Tracer $E(\rho(T))$ et $U(\rho(T))$ en fonction de T pour differentes valeurs de D

Second cas $(ii) b_1 = b_2 = b$

Prendre une valeur positive J_z (0.5) et aussi une valeur negative (-0.5)

Tracer $E(\rho(T))$ et $U(\rho(T))$ en fonction de T et b pour differentes valeurs de D

Let us consider here, the Ising chain with DM interaction in a external magnetic field. Fig.1 demonstrates the LQU behavior versus D_z and uniform magnetic field B_z. It is clear that the LQU is an increasing function of DM interaction D_z. Consequently, we show that the LQU can be enhanced by introducing the DM interaction. Similarly, a same behavior is observed in Fig.2 is that the LQU always increases in terms of DM interaction D_z versus inhomogeneity for magnetic field values b_z. It is noted that our result is in agreement with recent works where they showed the quantum correlations are enhanced by increasing the DM interaction[32]. Nevertheless, in a strong degree of inhomogeneous magnetic field, the spin system is known a level-crossing (white line) that occur in around of a quantum critical point ($b_{zc} = 0.9$) and consequently, we can say that the Ising model undergoes in a quantum phase transition. An another aspect slightly different is depicted in Fig.3 which the thermally LQU is plotted versus D_z. One observes always a level-crossing when an increasing temperature and secondly the LQU behavior versus DM interaction remains almost constant with weak D_z. But, the LQU presents a sudden change in a $D_z = 0.75$ above which the LQU vanishes.

3.0.2 XXX spin model (J_x= J_y= J_z= J)

We now analyze XXX model where the J_x, J_y and J_z are assumed to equal to J. Fig.4 represents the LQU variation versus D_z and B_z. It is found that the increasing of D_z interaction leads to the decreasing of the LQU, and also increasing magnetic field B_z implies to enhance the LQU. However,Fig.5 demonstrates nonuniform magnetic field b_z affects negatively in the LQU such that increasing b_z will decrease LQU. Therefore,Fig.6

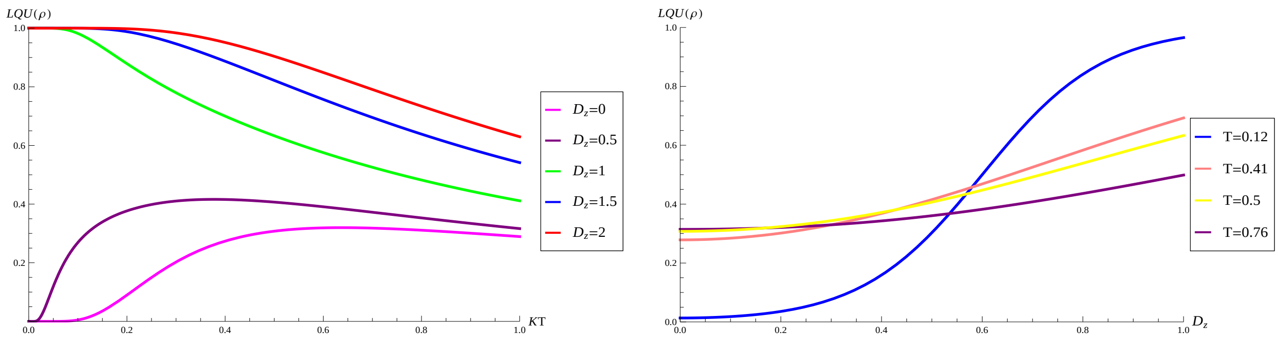

Figure 1: fig9:Model Ising jz=0.5,fig10:Model Ising1 jz=0.5,fig11:Model Ising3D jz=0.5,fig12:Model Ising3D jz=-0.5

shows thermal LQU versus different D_z values whose the thermal LQU decrease with increasing D_z parameter for $T > T_c$ where $T_c =$ is the critical temperature. Besides, when the temperature is less than at $T = T_c$, the LQU takes zero value and we can against deduces that the critical temperature T_c is also affected by D_z parameter in which the T_c will increase when parameter D_z increases .

3.0.3 XYZ spin model ($J_x \neq J_y \neq J_z$)

In order to see the brought novelty, if here we consider that $Jx \neq Jy \neq Jz$ which an another behavior can be observed from Fig.7 is that a level-crossing (white line) occurs between the fundamental and excited state of Heisenberg system. one notices that also the level crossing takes place on the range of values of between $D_z \in [0.47, 0.65]$. On the other hand, we depict the LQU behavior versus D_z and b_z parameters (Fig.8). it is seen that again a level-crossing (white line) between the ground and excited states is displayed and in this case takes places as circular form. Note that, the results obtained in Heisenberg XYZ model are similar to those developed in recent work describing the ground energy behavior of Heisenberg XYZ Hamiltonian [48]. In addition, thermal LQU is reported as function of parameter D_z (Fig.9) such as the thermal LQU increases when parameter D_z increases unlike what we found in section **4.0.2**. Besides, this sudden change of thermal LQU can be attributed in the consideration of the $J_x \neq J_y \neq J_z$, so we say that this case the DM interaction tend to enhance the thermal LQU.

Note that, all sudden changes of quantum correlations observed in several Heisenberg systems are generally explained by a quantum phase transition occuring [50, 51, 52, 53, 54, 55]. Similarly, the sudden change presented in LQU is due to the non-analyticity of the derivative of the LQU in the changing points [56]. We found that the DM interaction plays a important role in detecting quantum phase transition in Heisenberg systems.

4 Conclusions

In summary, we have studied the LQU in two-qubit Heisenberg XYZ system in presence of DM interaction and uniform and inhomogeneous magnetic field. Firstly, we found that the LQU measurement is affected by (D_z ,B_z,b_z) parameters. Furthermore, we have showed that the DM interaction play different behaviors in Heisenberg systems where it improve LQU measurement in model Ising but in XXX model degrade the LQU

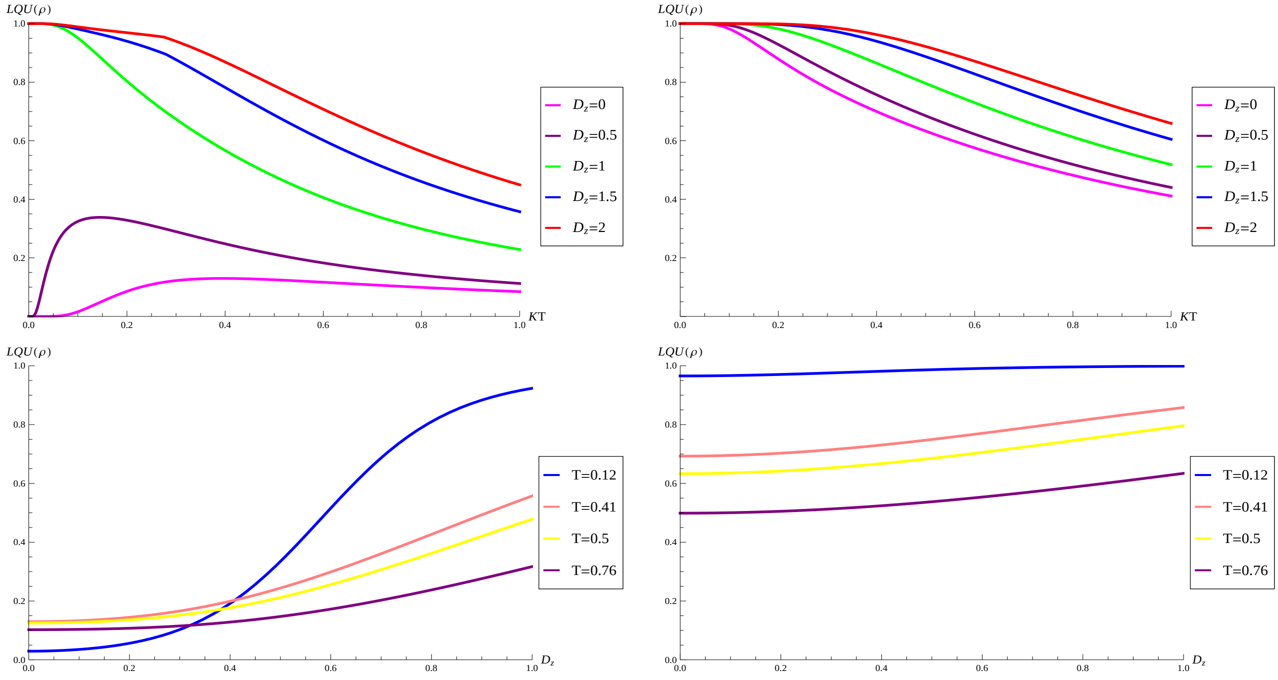

Figure 2: fig13:Model XXX jz=0.1, j=0.1,fig14:Model XXX jz=0.5,fig15:Model XXX-2D jz=0.1 j=0.10,fig16:Model XXX2D jz=0.5

measurement.

On other hand, the DM interaction plays a important role is to detect quantum phase transition in Heisenberg systems. However, it is shown that we can use LQU measurement to exhibit quantum phase transition of the Ising model and XYZ model, but in XXX model no phase transition takes places. Note that the XX, XXZ models are also treated, but their results are respectively similar to Ising model, XXX model.

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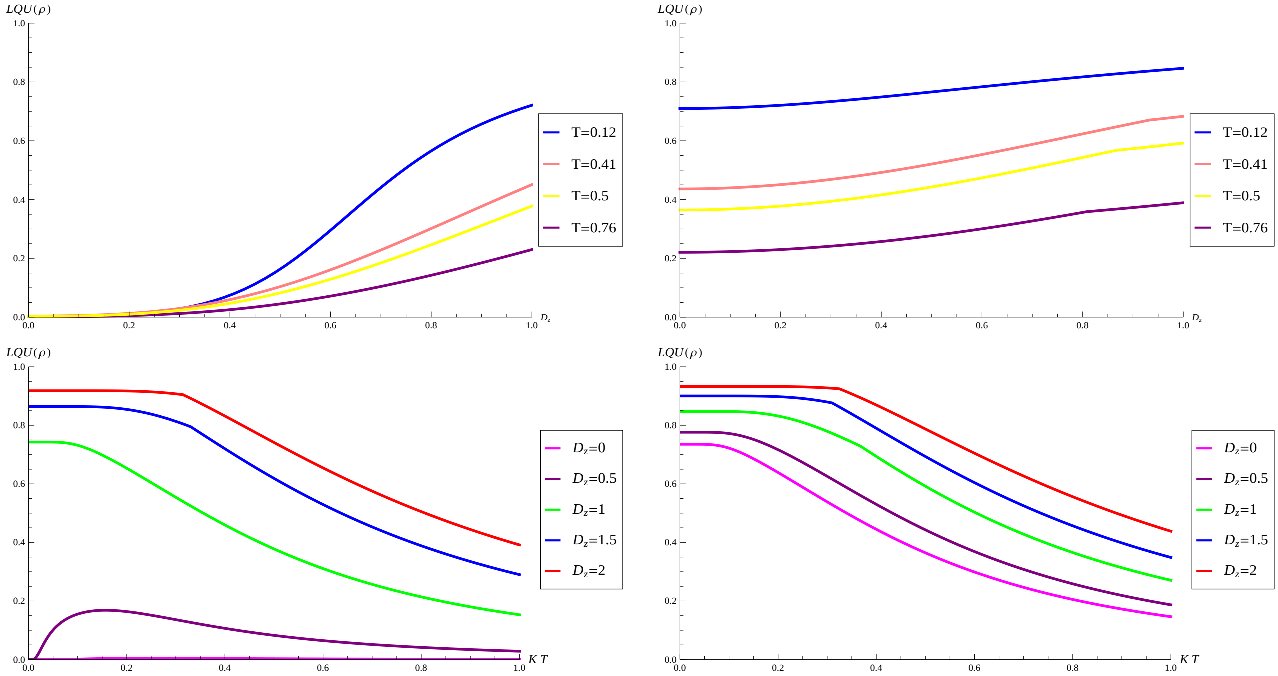

Figure 3: fig30:Model XY2D j=0.1 jz=0,fig31:Model XY2D jz=0 J = 0.5,fig32:Model XY-2D j=0.1 jz=0,fig33:Model XY-2D jz=0 J = 0.5,fig34:Model XY-2D jz=0,fig35:Model XY-3D j=0.1 jz=0

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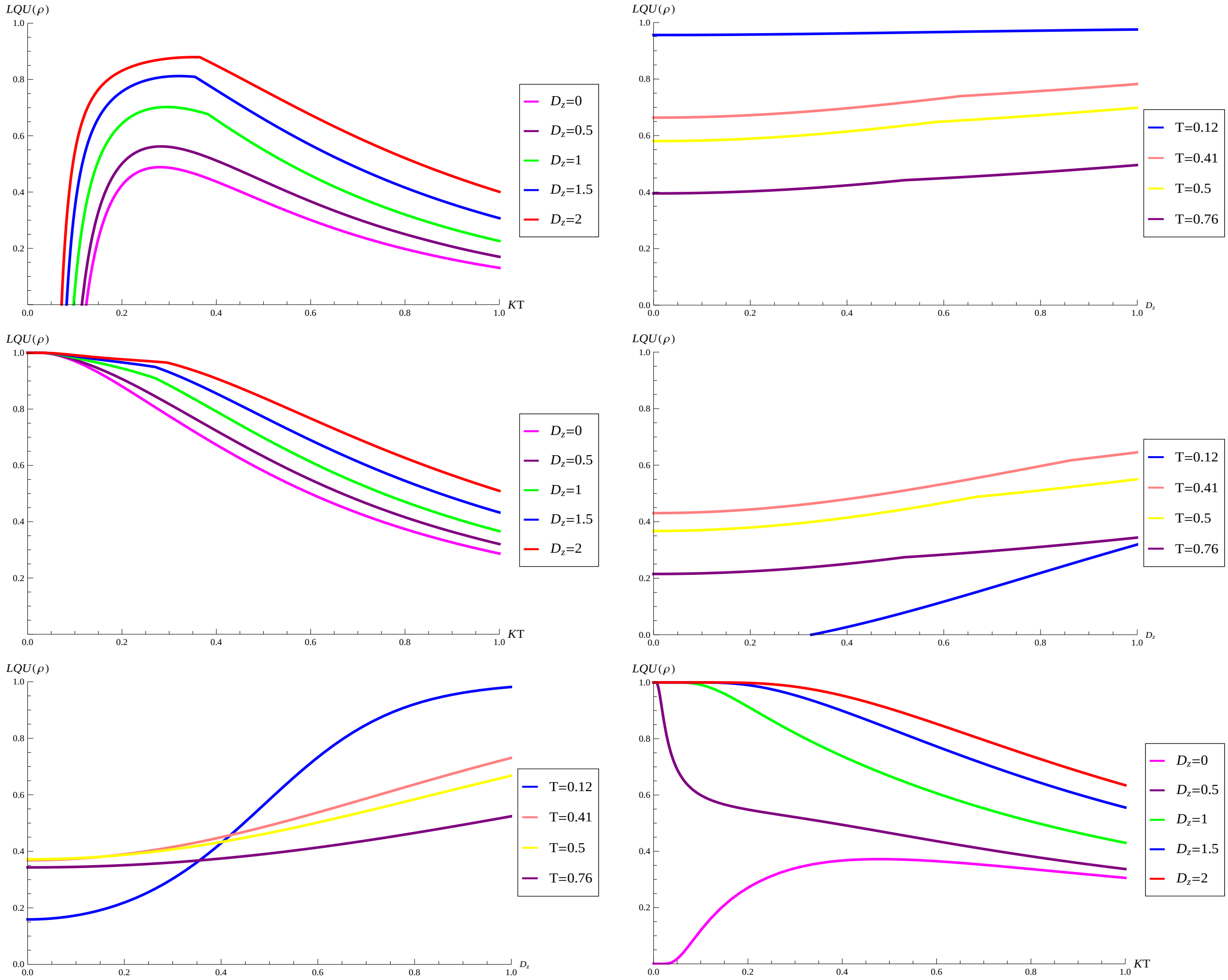

Figure 4: fig20:Model XXZ jz=-0.1 jx=jy=0.6,fig21:Model XXZ2D jz=0.1 jx=jy=0.6,fig22:Model XXZ-2D jz=0.1 jx=jy=0.6, fig23:Model XXZ-2D jz=-0.1 jx=jy=0.6,fig24:Model XXZ-2D jz=0.5,fig25:Model XXZ2D jz=05

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