PRA (Submission No: AU 11419)

Title: Dynamics Gaussian one-way steering in optomechanical systems

We would like to thank the referee for his constructive remarks, comments and suggestions.
We did as best as we could to take into account all his comments.
The important corrections, suggested by the referee, are marked in red in the revised version.

List of changes

concerning the comment

The presented analysis is theoretically sound. As a whole, the manuscript is, however, mediocre in terms of presentation (language, clarity) and content.

our reply

A special attention was devoted to the presentation and the clarification of the purpose and the results of this work.

concerning the comment

From the outset the authors fail to explain what the concrete goal of their work is.$\cdots\cdots\cdots$ Why do they deem an optomechanical implementation of EPR steering interesting? Is it the fundamental or the applied aspects? If it is the latter, which applications do they have in mind?

our reply

We thank the referee for this remark. This paper is essentially devoted to the study of one-way steering in an realistic optomechanical system which, in principle, can be implemented experimentally. This is a modest contribution to point out that it is possible to detect one-steering behavior between two mechanical modes. This point is discussed in the revised version.

concerning the comment

In the introduction, the authors explain the concept of steering by saying that LMCC can be used to steer a quantum state nonlocally. This is not very helpful.

our reply

To avoid any misunderstanding and confusion, the introduction has been rewritten to take into account all the remarks and the suggestions of the referee.

concerning the comment

(i) Section 2 should be shortened considerably. The applied approximations have been discussed extensively in the literature and there is no reason to reproduce them here.(ii) Additionally, I see no reason to apply the rotating wave approximation, at least not to get numerical results.

our reply

(i) We have reduced the size of this section.

(ii) This approximation simplifies considerably the numerical calculations and it is helpful to get closed analytical expressions for the covariance matrix.

concerning the comment

(i) In section 3, the optomechanical parameters are taken from reference [41], which was published in 2007. This publication is not at all "recent" in the field of optomechanics. (ii) Additionally, the thermal occupation numbers assumed to produce the plots are completely unrealistic for a 1MHz mechanical resonator. (iii) The conclusion that demonstration of EPR steering using optomechanical systems is feasible cannot be drawn based on these parameters.

our reply

(i)- Indeed, the reference [S. Gröblacher *et al*,Nature 460, 724 (2009).] is not a recent publication. The experimental results for this kind of optomechanical system are rare.

(ii)- Concerning the point "*the mean thermal photons numbers* $n_{th,j}$ *which have been used to obtain our results, are unrealistic for 1MHz mechanical resonator and can not be used to do an experimental test*", we notice that the values of $n_{th,1}$ and $n_{th,2}$ are of the same order as in [S. Huang and G. S. Agarwal, New J. Phys. 11, 103044 (2009)] where the mechanical frequency $\omega_m = 2\pi \times 947 \times 10^3$Hz.

(iii)- We believe that the EPR steering can be detected in such optomechanical system and only the experiment can determine the appropriate values of the physical parameters. The choice of the parameters used in this paper is dictated by the available experimental data in the literature.

concerning the comment

The authors claim that figure 2 and 3 show that the steerability is bounded by entanglement. As shown in [9] this must indeed be the case, but only if entanglement is measured in terms of Rényi-2-entropy, not logarithmic negativity (logneg)! The claim is therefore invalid!

our reply

We have replaced the logarithmic negativity E_N by Gaussian Rényi-2 entropy $\mathcal{E}_2$ to quantify entanglement. We have found that Gaussian steering is always upper bounded by Gaussian Rényi-2 entropy in agreement with the result of Kogias *et al* in [PRL. 114, 060403 (2015)].

concerning the comment

The observation that the logneg cannot detect EPR steerability is trivial given that the logneg is defined completely symmetric with respect of exchange of parties (as is evident from eq (35)). This should not be more than a sidenote and not a selling point of the manuscript in the abstract.

our reply

We agree with this remark. This is corrected.

concerning the comment

The plots in figures 2 and 3 should be scaled such that the axis labels align correctly for better comparability.

our reply

The same scaling is used in the revised version.

concerning the comment

The labeling of the modes a^{in} does not match the description used in the text.

our reply

This is corrected (see Figure 1 in the revised manuscript).

concerning the comment

This lack of a clear goal makes it difficult to judge the manuscript's quality and if the author's actually achieved their goal. To be precise, the manuscript presents a protocol to generate a mechanical resource state for EPR steering; it does not present a complete protocol to demonstrate steering using optomechanical systems. Although the authors talk about homodyne detection at first (figure 1), measurements are completely neglected in the rest of the manuscript. This may sound like a trivial objection, but the mechanical quantum state can only be inferred using measurements of the output light. This has two effects: It introduces additional noise (which I think should not be a problem), but it also makes it harder to argue that the measured steering is actually mechanical steering and not only due to the TMS input light (which in the current setup would also be detected by the homodyne detection). In this respect I find the presented analysis severely incomplete (it might be easy to rescue, however).

our reply

The measurement of Gaussian steering between the mechanical modes can be obtained by adopting the strategy developed by Vitali *et al* in [PRL 98, 030405 (2007)]. In fact, one has to measure the elements of the covariance matrix. In this order, for each cavity Fabry-Perot, one considers a second auxiliary cavity formed by the movable mirror and another transmitting mirror (see the figure below). Using the results of the section 2, it is simple to verify that the dynamics of the operators $\tilde{c}_j$ and $\tilde{b}_j$ are governed by the following equations

$$\delta\dot{\tilde{b}}_j = -\frac{\gamma_j}{2}\delta\tilde{b}_j + G_j\delta\tilde{c}_j + \sqrt{\gamma_j}\tilde{b}_j^{in}, \quad (1)$$

$$\delta\dot{\tilde{c}}_j = -\frac{\kappa_j}{2}\delta\tilde{c}_j - G_j\delta\tilde{b}_j + \sqrt{\kappa_j}\tilde{c}_j^{in}. \quad (2)$$

The only non-vanishing correlation function of the operators c_j is $\langle c_j^{in}(t)c_j^{in\dagger}(t')\rangle = \delta(t-t')$. Applying the adiabatic approximation on the resulting $\delta\tilde{c}_j$ (Eq. (2)), one has $\delta\tilde{b}_j = -\frac{\kappa_j}{2G_j}\delta\tilde{c}_j + \frac{\sqrt{\kappa_j}}{G_j}\tilde{c}_j^{in}$. In addition, using the Input-Output relation given by $\tilde{c}_j^{out} = -\tilde{c}_j^{in} + \sqrt{\kappa_j}\delta\tilde{c}_j$, one gets

$$\delta\tilde{b}_j = -\frac{\sqrt{\kappa_j}}{2G_j}\tilde{c}_j^{out} + \frac{\sqrt{\kappa_j}}{2G_j}\tilde{c}_j^{in}. \quad (3)$$

Equation (3) shows that the output laser field of the j^{th} auxiliary cavity, measured by the homodyne detection, gives a direct measurement of the j^{th} mirror dynamics. This gives the experimental way to measure the entries of the correlation describing the two mechanical modes A and B.

As this result was discussed in Vitali *et al* in [PRL 98, 030405 (2007)], We believe that its reproduction here is unnecessary but we added the following sentence (the end of the subsection 3.1):

It important to stress that the readout-scheme proposed by Vitali *et al* in [PRL 98, 030405 (2007)] can be used to measure the entries of the correlation matrix between the two mechanical modes and to fully characterize the steerability in optomechanical system.